Mathematics Methods 3 and 4
Test 2 Calculator Free

Name: ………………………………………………

Teacher: Mrs Martin   Dr Moore   Mr Smith

Time Allowed : 30 minutes

Marks   /30

Materials allowed: Formulae Sheet provided.
Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

Question 1   [4, 2, 2 = 8 marks]

Determine the following:

(a) $\int \left(e^{2x} + \sqrt{x} + \pi\right)dx$

(b) $\dfrac{d}{dx}\displaystyle\int_{x}^{3}\dfrac{t^2}{4}\,dt$

(c) $\dfrac{d}{dx}\displaystyle\int_{0}^{x^2}\sqrt{1+t^2}\,dt$

Question 2   [3, 4 = 7 marks]

Evaluate

(a) $\displaystyle\int_{0}^{2}\dfrac{3}{(2x+1)^4}\,dx$

(b) $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}2\sin 2x\,dx$

Question 3   [1, 3 = 4 marks]

Given $\int_{0}^{-3} f(x)dx = 1$ and $\int_{0}^{2} f(x)dx = -5$, find

(a) $\int_{-3}^{2} f(x)dx$ (b) $\int_{0}^{2} [3f(x) - 4]dx$

Question 4   [5 marks]

Given $\dfrac{dy}{dx} = ae^{x} + 1$ and when $x = 1$, $\dfrac{dy}{dx} = 3$ and $y = 2$

Find the value of y when $x = 0$.

Question 5   **[1, 1, 2, 2 = 6 marks]**

Given $h(x) = \int\limits_{0}^{x} \cos(2t)dt$, determine

a) $h'(x)$

b) $h'\left(\dfrac{\pi}{2}\right)$

c) $h\left(\dfrac{\pi}{2}\right)$

d) the equation of the tangent to the curve $h(x)$ at $x = \dfrac{\pi}{2}$

Teacher: Mrs Martin   Dr Moore   Mr Smith

Time Allowed: 20 minutes

Marks   /27

Materials allowed: Formulae Sheet provided. Classpad, calculators, 1 A4 page of notes, one side.
Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

Question 1   [2, 2, 2, 2 = 8 marks]

The acceleration (m/s^2) of a particle moving in a straight line is given by $a = 2t - 4$. The particle's initial velocity is 3 m/s. Its initial displacement from the origin is -15 m.

(a)   Find the expression for the particle's velocity at any time.

(b)   Find the time(s), if any, when the particle comes to rest.

(c) Find its displacement when $t = 3$

(d)   Find the distance travelled in the first 3 seconds.

Question 2   **[3, 6 = 9 marks]**

Consider the functions: $f(x) = x(5-x)$ and $g(x) = x(x-3)$

(a) Write down an integral which when evaluated will determine the area trapped between the
    two functions and calculate the area.

(b) Within the area trapped between the two functions a vertical line is drawn, intersecting $f(x)$ at Point P
    and intersecting $g(x)$ at Point Q.

   (i)   Show use of calculus to find the value of x for which the length of line segment PQ is a maximum.

   (ii)   Use the second derivative test to show that this value of x does indeed produce a maximum value.

   (iii) State the maximum length possible.

Question 3 [2, 3 = 5 marks]

The ratio of the radius (r) to the height (h) is 5:3 for a specific cone.

(a) Show that the volume of the cone is given by $V = \dfrac{25\pi h^3}{27}$

(b) Use the method of small change to find the approximate increase in the volume of the cone if the height changes from 5 cm to 5.02 cm.

Question 4 [2, 3 = 5 marks]

The cost, $C(x)$ ($1000s) of manufacturing a product is given by $C(x) = 45 + 65x$. The revenue, $R(x)$, is given by the function $R(x) = 100x - 2.5x^2$. The manufacturer can only make between 2 and 10 products per week.

Find

(a) a simplified expression for the Profit if x units are made and sold.

(b) the minimum and maximum profit possible each week.